365. Proposed by C. N. SCHMALL, New York City.

Show that the area inclosed by each of the following three curves is equal to the circle of radius a; viz., πa^2.

(1) $a^2x^2 = y^3(2a - y)$, (2) $a^2 - x^2 = (y - mx^2)^2$, (3) $(xy + c + bx^2)^2 = x^2(a^2 - x^2)$.

I. Solution by A. M. Harding, University of Arkansas.

If we change these equations to parametric forms we obtain

(1) $x = 4a \cos^3 t \sin t, \quad y = 2a \cos^2 t,$

(2) $x = a \sin t, \quad y = ma^2 \sin^2 t + a \cos t,$

(3) $x = a \sin t, \quad y = \dfrac{a^2 \cos t - a^2 b \sin t - c \csc t}{a}.$

Hence,

(1) $\text{Area} = \int y\, dx = \int_0^{\pi} 8a^2(4 \cos^6 t - 3 \cos^4 t)dt = \pi a^2;$

(2) $\text{Area} = \int y\, dx = \int_0^{2\pi} (ma^2 \sin^2 t + a \cos t)a \cos t\, dt = \pi a^2;$ and

(3) $\text{Area} = \int y\, dx = \int_0^{2\pi} (a^2 \cos t - a^2 b \sin t - c \csc t) \cos t\, dt = \pi a^2.$

II. Solution by H. C. Feemster, York College, Nebraska.

$$A_1 = \int_0^{2a} \int_{-(y/a)\sqrt{2ay - y^2}}^{(y/a)\sqrt{2ay - y^2}} dydx = \left[-\frac{3a^2 + ax - 2x^2}{3} \sqrt{2ax - x^2} + a^2 \operatorname{vers}^{-1} \frac{x}{a} \right]_0^{2a} = \pi a^2;$$

$$A_2 = \int_{-a}^{a} \int_{mx - \sqrt{a^2 - x^2}}^{mx + \sqrt{a^2 - x^2}} dxdy = \left[x\sqrt{a^2 - x^2} + a^2 \sin^{-1} \frac{x}{a} \right]_{-a}^{a} = \pi a^2;$$

and

$$A_3 = \int_{-a}^{a} \int_{-\sqrt{a^2 - x^2} - (c/x) - bx}^{\sqrt{a^2 - x^2} - (c/x) - bx} dxdy = \left[x\sqrt{a^2 - x^2} + a^2 \sin^{-1} \frac{x}{a} \right]_{-a}^{a} = \pi a^2.$$

NUMBER THEORY.

218. Proposed by ELIJAH SWIFT, University of Vermont.

If p is prime > 3 show that

$$\sum_{a=1}^{a=p-1} \frac{1}{a^2} \equiv 0 \pmod{p}. \tag{1}$$

I. Solution by Tracy A. Pierce, Berkeley, Cal.

In (1), we may replace 1 by a^{p-1}, since $a^{p-1} \equiv 1 \pmod{p}$. We then have

$$\sum_{a=1}^{a=p-1} a^{p-3} \equiv 0 \pmod{p}.$$

But it is well known that the sum of like powers of the numbers 1, 2, 3, $\cdots$, $p-1$ is divisible by p if the power is not a multiple of $p-1$; hence, the theorem is proved.

As a generalization of the congruence above, we may state

$$\sum_{a=1}^{a=p-1} \frac{1}{a^k} \equiv 0 \pmod{p}$$

if k is not a multiple of $p-1$.

II. Solution by the Proposer.

$$\sum_{a=1}^{a=p-1} \frac{1}{a^2} \equiv \sum_{a=1}^{a=p-1} a^2 \pmod{p}.$$

But

$$\sum_{a=1}^{a=p-1} a^2 \equiv A_1{}^2 - 2A_2$$

where

$$A_1 = \sum_{a=1}^{p-1} a, \qquad A_2 = \sum_{\substack{a_1=1\\a_2=1}}^{p-1} a_1 a_2, \qquad a_1 \neq a_2.$$

In Bachmann's *Niedere Zahlentheorie*, Vol. I, page 155, it is proved that A_1 and A_2 are divisible by p, whence the theorem.

NOTES AND NEWS.

Edited by W. D. Cairns.

Professor Andrew W. Phillips, of Yale University, died on January 20, 1915. He was joint author of Phillips and Fisher's *Geometry*.

Professor R. M. Barton of the University of New Mexico has been appointed professor of mathematics in Lombard College.

Professor W. H. Roever, representative of Washington University on the editorial staff of the Monthly, was elected to honorary membership in the Society of Phi Beta Kappa by the Washington University Chapter, which was installed last year.

"The arithmetic mean as approximately the most probable value *a posteriori* under the gaussian probability law," by Edward L. Dodd, is the title of a pamphlet published by the University of Texas as its January, 1915, *Bulletin*.

The February number of the *Proceedings of the National Academy of Sciences* contains two mathematical articles. The first is by Professor E. J. Wilczynski and bears the title "Conjugate systems of space curves." The second is due to